The UNNS Gauge Path Integral (UGPI): Recursive Gauge Fields and Wilson Loops

UNNS Research Notes

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Abstract

We extend the UNNS Path Integral Protocol into the *Gauge Path Integral* framework. Here, recursion coefficients act as gauge connections, echo residues become discrete field strengths, and path integrals are performed over equivalence classes of gauge-related nests. Wilson loop analogs measure curvature of recursion, providing a direct bridge between UNNS, Maxwell theory, and Yang–Mills gauge fields.

Contents

1 Gauge Transformations in UNNS

Definition 1.1 (Gauge Transformation)**.** *Let $\{a_e\}$ be recursion coefficients assigned to edges of a UNNS mesh. A gauge transformation is a local rescaling*

$$a_e \mapsto g(v_{start})\, a_e\, g(v_{end})^{-1},$$

where g : Vertices $\to G$ is a map into a group G (e.g. $U(1)$ or $SU(n)$).

Remark 1.2. *This parallels standard lattice gauge theory, where connections live on edges and gauge transformations act at vertices.*

2 Gauge Action Functional

Definition 2.1 (Field Strength). *For a cycle (closed path) $C = e_1 e_2 \cdots e_k$, define the holonomy*

$$U(C) = a_{e_1} a_{e_2} \cdots a_{e_k}.$$

The gauge-invariant field strength is

$$F(C) = U(C) - I.$$

Definition 2.2 (Gauge Action). *The UNNS gauge action is*

$$S_G[\{a_e\}] = \sum_{C \in \mathcal{C}} Tr(F(C)^\dagger F(C)),$$

where $\mathcal{C}$ is the set of elementary cycles in the recursion mesh.

3 Partition Function and Wilson Loops

Definition 3.1 (Gauge Partition Function). *The path integral over recursion gauge fields is*

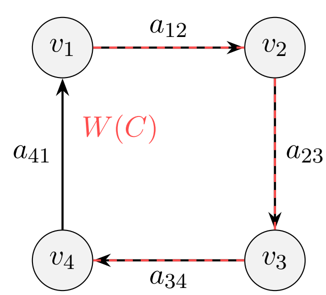
$$Z = \int \mathcal{D}a_e \, e^{iS_G[\{a_e\}]}.$$

Definition 3.2 (Wilson Loop Analog). *The expectation value of a loop observable is*

$$\langle W(C) \rangle = \frac{1}{Z} \int \mathcal{D}a_e \; Tr(U(C)) \, e^{iS_G[\{a_e\}]}.$$

Theorem 3.3 (Gauge Invariance). *Both Z and $\langle W(C) \rangle$ are invariant under vertex gauge transformations.*

4 Diagrammatic Overview



5 Applications

5.1 Mathematics

- Provides discrete analogs of curvature and holonomy.
- Embeds recursion into lattice gauge theory framework.

5.2 Physics

- Maxwell fields: $G = U(1)$ recursion gauge group.
- Yang–Mills: $G = SU(n)$, capturing non-abelian recursion dynamics.

5.3 Computation

- Wilson loops as diagnostics for recursion stability.
- Gauge redundancy suggests error-correcting interpretations.

6 Conclusion

The UNNS Gauge Path Integral unifies recursion with gauge theory, treating coefficients as edge connections and holonomies as curvature. This elevates UNNS into a discrete gauge-theoretic field framework, with direct analogues to Maxwell and Yang–Mills dynamics.